

MIDPOINT, MEDIAN, & RIGHT BISECTOR

LEARNING GOALS

Students will:

- Learn how to find the midpoint of a line segment.
- Learn how to find the median of a triangle.
- Learn how to find a right bisector.

MIDPOINT

The...

Slope	A measure of the steepness of a line	Formula $\text{Slope} = \frac{\text{rise}}{\text{run}}$
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Can be used to find the...

Midpoint	A point that divides a line segment into two equal line segments	Formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
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How might you use the calculation of a midpoint in your lives?

EXAMPLE 1: FIND A MIDPOINT

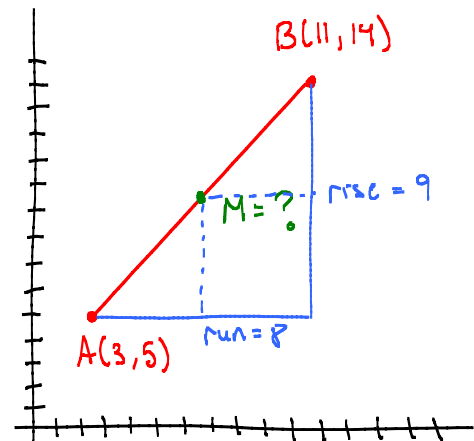
There are two hospitals located at coordinates A (3, 5) and B (11, 14). The city wants to build a new ambulance station halfway between the two hospitals. Determine the coordinates of this location.

Method 1: Calculate the slope

$$\begin{aligned} \text{rise} &= y_2 - y_1 & \text{run} &= x_2 - x_1 \\ &= 14 - 5 & &= 11 - 3 \\ &= 9 & &= 8 \end{aligned}$$

Therefore,

$$\begin{aligned} M = (x, y) &= \left(x_1 + \frac{\text{run}}{2}, y_1 + \frac{\text{rise}}{2} \right) \\ &= \left(3 + \frac{8}{2}, 5 + \frac{9}{2} \right) \\ &= (3 + 4, 5 + 4.5) \\ &= (7, 9.5) \end{aligned}$$



Method 2: Use the Midpoint formula.

$$\begin{aligned} M = (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{3 + 11}{2}, \frac{5 + 14}{2} \right) \\ &= \left(\frac{14}{2}, \frac{19}{2} \right) &= (7, 9.5) \end{aligned}$$

Midpoint Median	A line segment joining a vertex of a triangle to the midpoint of the opposite side.
Vertex	A point that represents the intersection of sides.

EXAMPLE 2: MEDIAN OF A TRIANGLE

Determine an equation for the median from vertex C for the triangle with vertices C(5, 2), A(-3, 3), and B(2, -5).

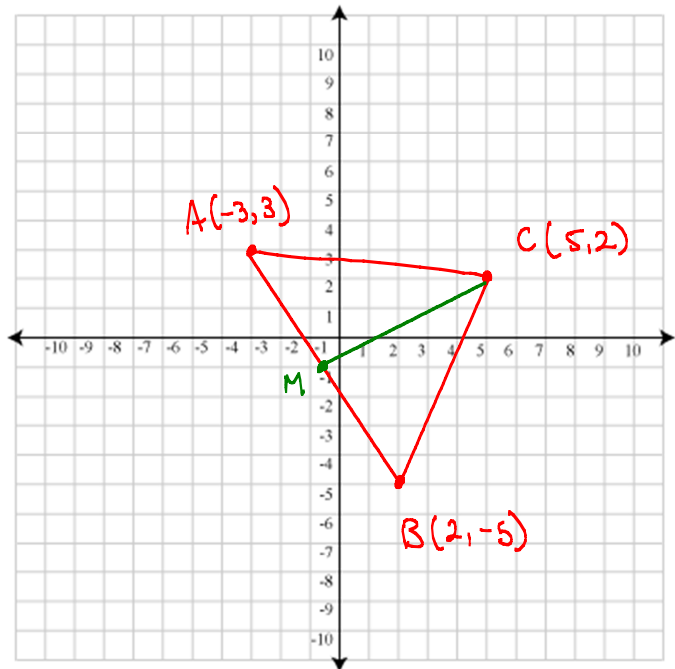
1) Find the coordinates of the midpoint between A + B.

$$\begin{aligned} (x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-3 + 2}{2}, \frac{3 + (-5)}{2} \right) \\ &= \left(-\frac{1}{2}, -\frac{2}{2} \right) \\ &= \left(-\frac{1}{2}, -1 \right) \end{aligned}$$

2) Now find the slope of the line segment CM.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-1)}{5 - (-\frac{1}{2})} \\ &= \frac{3}{\frac{11}{2}} \\ &= 3 \times \frac{2}{11} \\ &= \frac{6}{11} \end{aligned}$$

$$y = \frac{6}{11}x - \frac{8}{11}$$



3) Use the point C (or M) to find the y-intercept using the equation of a line.

$$y = \frac{6}{11}x + b$$

$$2 = \frac{6}{11}(5) + b$$

$$2 = \frac{30}{11} + b$$

$$2 - \frac{30}{11} = b$$

$$\frac{22}{11} - \frac{30}{11} = b$$

$$b = -\frac{8}{11}$$

Right Bisector	The line that passes through the midpoint of a line segment and intersects it at a 90° angle.
Equidistant	Equally Distant.

EXAMPLE 3: EQUATION OF A RIGHT BISECTOR

Two schools are located at points $P(-1, 4)$ and $Q(7, -2)$ on a town map. The school board is planning a new sports complex to be used by both schools. The board wants to find a location **equidistant** from the two schools. Use an equation to represent the possible locations for the athletic complex.

A point can be equidistant from the schools anywhere along the right bisector.

- 1) Find the slope of the line PQ
 - Perpendicular lines have slopes that are the negative reciprocals of each other.
- 2) The right bisector passes through the midpoint of PQ
 - Provides a point along the line required to find the y -intercept.
- 3) Find the y -intercept using the slope and the midpoint.

