

VECTOR OPERATIONS

LEARNING GOALS

Students will:

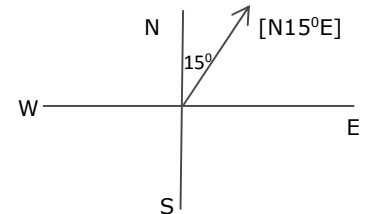
- Describe vectors graphically and mathematically
- Add and subtract vectors using mathematical methods

VECTORS

1. Write a definition for a vector:

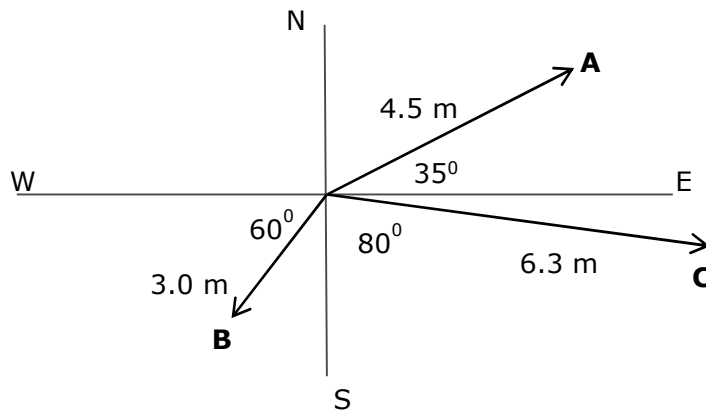
STEPS TO DRAWING VECTORS

1. Draw an x,y axis.
2. Measure the angle starting from the x-axis or based on the letter direction provided. A direction like [N15°E] is read "north, fifteen degrees east". This direction is found by starting at the north line and measuring 15° towards the east. Note: [N15°E] can also be written as [15° E of N]



WRITING VECTORS

1. Here are some vectors drawn in a horizontal plane (not to scale):



- a) State each vector in the form *magnitude [direction]* (e.g. 5.0 m [E25°N]):

A 4.5 m [E 35° N]

B 3.0 m [W 60° S]

C 6.3 m [S 80° E]

- b) Each vector has components in two of the main directions. For example, **A** has a north component and an east component as it is in between north and east. Sketch the components of each vector and use trigonometry to find the magnitude of each component.

A $A_x = 4.5 \cos 35^\circ = 3.7 \text{ m [E]}$ $A_y = 4.5 \sin 35^\circ = 2.6 \text{ m [N]}$

B $B_x = 3.0 \cos 60^\circ = 1.5 \text{ m [W]}$ $B_y = 3.0 \sin 60^\circ = 2.6 \text{ m [S]}$

C $C_x = 6.3 \sin 80^\circ = 6.2 \text{ [E]}$ $C_y = 6.3 \cos 80^\circ = 1.1 \text{ m [S]}$

ADDING VECTORS

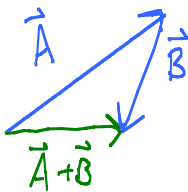
3. What is the rule for adding vectors? State it in words:

Add vectors tip to tail.

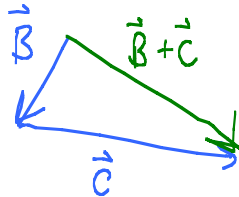
The resultant is drawn from the initial point to the final point.

4. Draw a vector diagram showing the addition of vectors

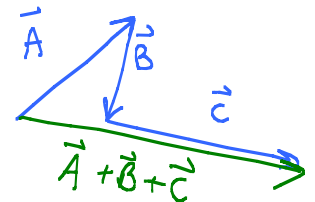
A + B



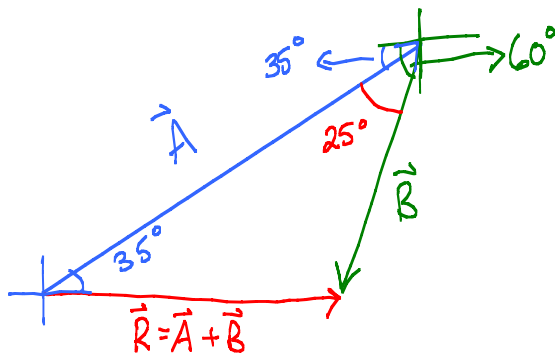
B + C



A + B + C



5. The first two examples above are triangles. Use the sine and cosine laws to solve for the unknown side of **A + B** (the side representing the sum). You will have to figure out the angle between the two known vectors first. (Answers: **A + B** = 2.2 m [E] directions rounded to nearest degree)



\vec{R} → resultant

$$\angle R = 60^\circ - 35^\circ = 25^\circ$$

Unless drawn to scale, do not assume the resultant is at an angle of 0° . Use what can be proven mathematically.

$$A = 4.5\text{m}$$

$$B = 3.0\text{m}$$

$$\angle R = 25^\circ$$

$$R = ?$$

$$R^2 = A^2 + B^2 - 2AB \cos \angle R$$

$$= 2.2\text{m}$$

$$\frac{R}{\sin \angle R} = \frac{B}{\sin \angle B}$$

$$\sin \angle B = \frac{B \sin \angle R}{R}$$

$$\angle B = 35^\circ$$

$$\therefore \vec{R} = 2.2\text{m [E]}$$

6. Repeat the additions you performed above by adding the components of the vectors. Also add all three vectors using the component technique. Draw a diagram for each one. (answer $\mathbf{A} + \mathbf{B} + \mathbf{C} = 8.5 \text{ m [E}8^\circ\text{S]}$)

A + B

$$\vec{R} = \vec{A} + \vec{B}$$

$$R_x = A_x + B_x = 3.7\text{m} + (-1.5\text{m}) \\ = 2.2\text{m [E]}$$

$$R_y = A_y + B_y = 2.6\text{m} + (-2.6\text{m}) \\ = 0\text{m}$$

$$\vec{R} = 2.2\text{m [E]}$$

A + B + C

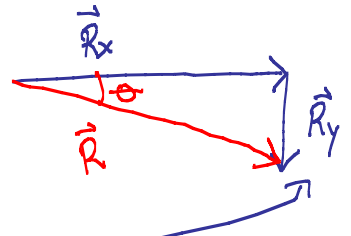
$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$R_x = A_x + B_x + C_x \\ = 3.7\text{m} + (-1.1\text{m}) + 6.2\text{m} \\ = 8.4\text{m [E]}$$

$$R_y = A_y + B_y + C_y \\ = 2.6\text{m} + (-2.6\text{m}) + (-1.1\text{m}) \\ = 1.1\text{m [S]}$$

$$R = \sqrt{R_x^2 + R_y^2} \\ = 8.5\text{m}$$

$$\tan \theta = \frac{R_y}{R_x} \quad \theta = 7.5^\circ \approx 8^\circ$$



$$\vec{R} = 8.5\text{m [E } 8^\circ \text{ S]}$$

7. Why do you think you were not asked to do $\mathbf{A} + \mathbf{B} + \mathbf{C}$ using sine and cosine law?

SUBTRACTING VECTORS

The easiest way to subtract vectors is to add the negative.

EXAMPLE:

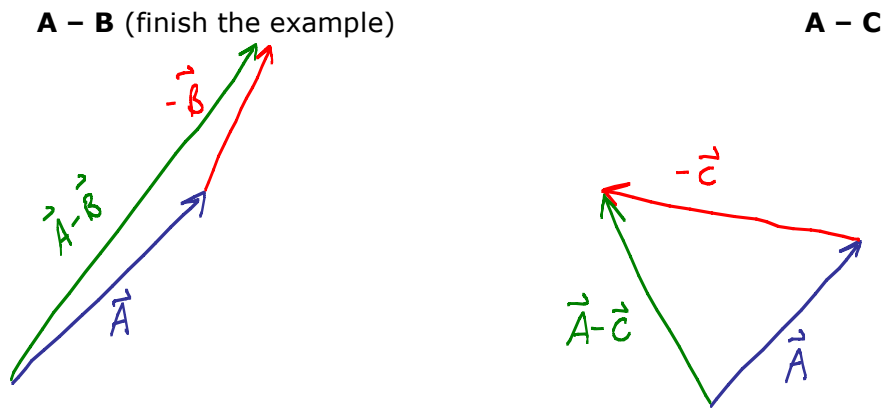
$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

$$\begin{aligned} \mathbf{A} - \mathbf{B} &= 4.5 \text{ m [E}35^\circ\text{N]} + (-3.0 \text{ m [W}60^\circ\text{S]}) \\ &= 4.5 \text{ m [E}35^\circ\text{N]} + 3.0 \text{ m [E}60^\circ\text{N]} \end{aligned}$$

Note that you are now adding \mathbf{A} to the negative or opposite direction of \mathbf{B} .

Now you can perform the addition as you did above.

8. Do the following. Draw a diagram for each one.



(answers: $\mathbf{A} - \mathbf{B} = 7.3 \text{ m [E}45^\circ\text{N]}$ $\mathbf{A} - \mathbf{C} = 4.5 \text{ m [W}56^\circ\text{N]}$)

PRACTICE

- Do the following vector operations on the given velocities. Use whichever method you prefer, but make sure you practice **both** methods.
 - 2.4 m/s [E] + 3.2 m/s [W]
 - 2.4 m/s [E] + 3.2 m/s [N]
 - 2.4 m/s [E] + 3.2 m/s [E45°S]
 - 2.4 m/s [E] + 3.2 m/s [W35°N]
 - 2.4 m/s [E35°N] + 3.2 m/s [W35°N]
 - 2.4 m/s [E] - 3.2 m/s [W35°N]
 - 2.4 m/s [E] + 3.2 m/s [W35°N] + 4.2 m/s [S10°W]
- Optional Online Activity: ExploreLearning.com has a vector Gizmo you can play with.

Answers:

- a) 0.8 m/s [W] b) 4.0 m/s [E53°N] c) 5.2 m/s [E26°S] d) 1.8 m/s [W83°N]
 e) 3.3 m/s [W78°N] f) 5.3 m/s [E20°S] g) 2.5 m/s [S22°W]