SIGNIFICANT DIGITS & UNCERTAINTIES

The number of significant digits in an answer to a calculation will depend on the number of significant digits in the given data, as discussed in the rules below. *Approximate* calculations (order-of-magnitude estimates) always result in answers with only one or two significant digits.

When are Digits Significant?

Non-zero digits are always significant. Thus, 22 has two significant digits, and 22.3 has three significant digits.

With zeroes, the situation is more complicated:

a. Zeroes placed before other digits are not significant; 0.046 has two significant digits.
b. Zeroes placed between other digits are always significant; 4009 kg has four significant digits.
c. Zeroes placed after other digits but behind a decimal point are significant; 7.90 has three significant digits.
d. Zeroes at the end of a number are significant only if they are behind a decimal point as in (c). Otherwise, it is impossible to tell if they are significant. For example, in the number 8200, it is not clear if the zeroes are significant or not. The number of significant digits in 8200 is at least two, but could be three or four. To avoid uncertainty, use scientific notation to place significant zeroes behind a decimal point:

$$8.200 \times 10^3 \text{ has four significant digits}$$

$$8.20 \times 10^3 \text{ has three significant digits}$$

$$8.2 \times 10^3 \text{ has two significant digits}$$

Significant Digits in Multiplication, Division, Trig Functions, etc.

In a calculation involving multiplication, division, trigonometric functions, etc., the number of significant digits in an answer should equal the least number of significant digits in any one of the numbers being multiplied, divided etc.

Thus in evaluating sin(kx), where k = 0.097 m^{-1} (two significant digits) and x = 4.73 m (three significant digits), the answer should have two significant digits.

Note that counted numbers have essentially an unlimited number of significant digits. As an example, if a hair dryer uses 1.2 kW of power, then 2 identical hairdryers use 2.4 kW:

1.2 kW {2 sig. dig.} x 2 {unlimited sig. dig.} = 2.4 kW {2 sig. dig.}

Significant Digits in Addition and Subtraction

When quantities are being added or subtracted, the number of *decimal places* (not significant digits) in the answer should be the same as the least number of decimal places in any of the numbers being added or subtracted.

Example:

5.67 J (two decimal places)
1.1 J (one decimal place)
0.9378 J (four decimal place)
7.7 J (one decimal place)

Keep Two Extra Digits in Intermediate Answers

When doing multi-step calculations, *keep at least one more significant digit in intermediate results* than needed in your final answer.

For instance, if a final answer requires two significant digits, then carry at least three significant digits in calculations. If you round-off all your intermediate answers to only two digits, you are discarding the information contained in the third digit, and as a result the *second* digit in your final answer might be incorrect. (This phenomenon is known as "round-off error.")

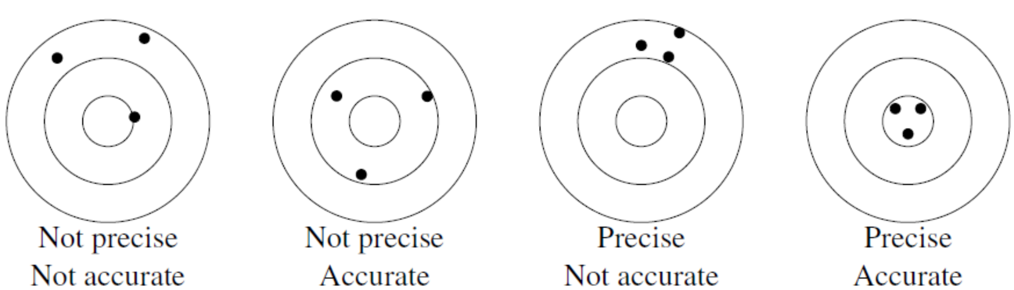
Error Analysis in Labs

Precision and Accuracy are two different things. A measured value is *precise* if

a) you were able to measure it to a large number of decimal places (e.g. 3.46 cm is more precise than 3 cm)
   OR
b) you took many trials of a particular value and the trials were all close together. For example:

Length (m)	Width (m)
34.12	25.14
34.51	24.79
34.36	25.69
34.45	25.99

In the table, the values of length are more precise than those of width. However, if the student measuring length accidentally held the ruler to measuring from 1.00 cm, then the values are not *accurate*. The distinction is also shown in the diagram below.



A measure of accuracy is % deviation:

$$\%deviation = \frac{theoretical\ value - experimental\ value}{theoretical\ value} x100$$

A measure of the precision of a list is % difference:

$$\%difference = \frac{maximum\ value - minimum\ value}{average\ value} x100$$

Uncertainty

Every measurement has a value of uncertainty. The last digit you record is always the uncertain digit and all measurements should have the uncertainty written down. The number of decimal places recorded and the uncertainty depends on the context of the lab (e.g. how precise you were realistically able to be)

ex. d = 3.25 cm ± 0.05 cm
h = 5.5 cm ± 0.2 cm

The number of decimal places in the measurement must match the number of decimal places in the uncertainty.

Sources of Uncertainty

DON'T SAY YOU MADE A MISTAKE**.** The following...

- rounding
- calculation errors
- something hit my equipment

are all mistakes. If you make a mistake, please fix it before you hand in your lab.

Legitimate uncertainties are things you try your best to minimize, but are unable to completely eliminate

Sometimes your measurement is limited by the instrument you are using – these are **instrumental uncertainties.**

The rules for instrumental uncertainty are as follows:

- For an <u>analogue instrument</u> (ex. metre stick) the uncertainty is half of the smallest division e.g. d = 3.42 cm ± 0.05 cm
- For a <u>digital instrument</u> (ex. electronic balance) the uncertainty is the smallest division e.g. m = 2.3 g ± 0.1 g

However, in many contexts, there are other uncertainties that are actually more important than the instrumental uncertainties. For example, when measuring the height a ball bounces, it's almost impossible to measure to half a millimeter because the ball goes so fast. In this case, you might record a value of 83.4 cm and the uncertainty might be ± 0.1 cm.