DATA ANALYSIS

LEARNING GOALS

Students will be able to take a set of data and

- determine the type of proportionality between the variables
- determine the equation that describes the data
- compare the equation from the data to the theoretical equation quantitatively

TYPES OF RELATIONSHIPS

GOAL: to be able to determine the proportionality between the variables in a physics formula

There are four primary types of relationships we will look at in Grade 12 University Preparation Physics: Linear ($y \propto x$), Exponential ($y \propto x^n$), Root ($y \propto \sqrt[n]{x}$ – square root or cubed root, fourth root etc) and Inverse ($y \propto \frac{1}{x^n}$). The symbol α is the Greek letter alpha and means "is proportional to"

You can tell the relationship between two variables simply by looking at an equation. For example, the equation $\vec{a} = \frac{\vec{F}_{net}}{m}$ shows that acceleration is linearly proportional to net force ($\vec{a} \propto \vec{F}_{net}$) and inversely proportional to mass ($\vec{a} \propto \frac{1}{m}$)

1. For each of the equations below, state the relationship between the given variables and express the relationship as a proportion.

   a. $\Delta\vec{d} = \frac{(\vec{v}_1 + \vec{v}_2)}{2}\Delta t$ Δd and Δt

   b. $\vec{F}_c = m\frac{v^2}{R}$ F and R

   c. $E_e = \frac{1}{2}kx^2$ E_e and x

FINDING THE PROPORTIONALITY AND EQUATION FROM DATA

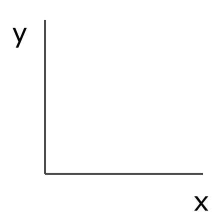
INTRO: WHAT DOES A PROPORTIONAL RELATIONSHIP LOOK LIKE?

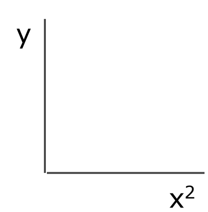
a) **KEY FACT: When two variables are proportional to each other, the graph of the variables will be a ____________________ _________________.**

Consider the equation $y=3x^2$.

b) Is y proportional to x? ______ How would a graph of y vs. x look? Sketch it below.

c) Is y proportional to x^2? ______ How would a graph of y vs. x^2 look? Sketch it below.





In this example the **proportion** is $y \propto x^2$ and the **equation** is $y=3x^2$. We say that **3 is the constant of proportionality** and gets the symbol **k**.

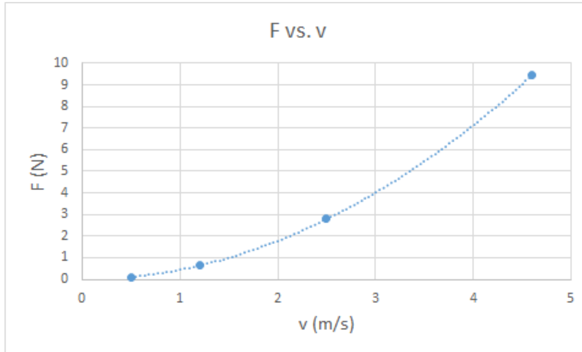
Check with your teacher

1 a) linear; $\Delta d \propto \Delta t$ b) inverse; $F \propto 1/R$ c) exponential; $E_e \propto x^2$

EXAMPLE ONE

Look at the data table and graph below.

v (m/s)	F (N)
0.5	0.11
1.2	0.64
2.4	2.57
4.8	10.27

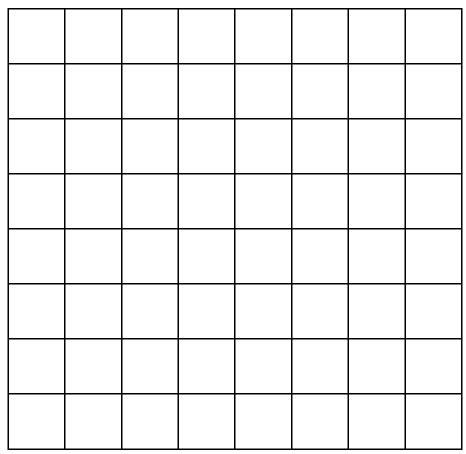


a) What type of relationship – linear, inverse, exponential or root - is shown by this data?

b) Now make a guess as to the actual proportionality. (i.e. is it $F \alpha v^2$? $F \alpha v^3$?) In a normal lab context, you will have a theoretical equation to base your guess off of.

c) Next, you will manipulate the data to represent the proportionality you guessed. Fill in the data table below. If you guessed that $F \alpha v^2$, then put v^2 in the x column and calculate v^2 to fill in the column. Graph your new data.

	F (N)
	0.11
	0.64
	2.57
	10.27



If your new graph is straight, this means that your guess was correct, because a straight line shows a proportional relationship. If your graph is NOT straight, make another guess and try again. (Make another table beside the first if necessary)

d) Based on your graph(s), what is the **proportionality** between F and v? ____________

e) You know that the equation for a straight line is y=mx+b. Use this to find the **equation** that relates F and v (replace x and y with v and F):

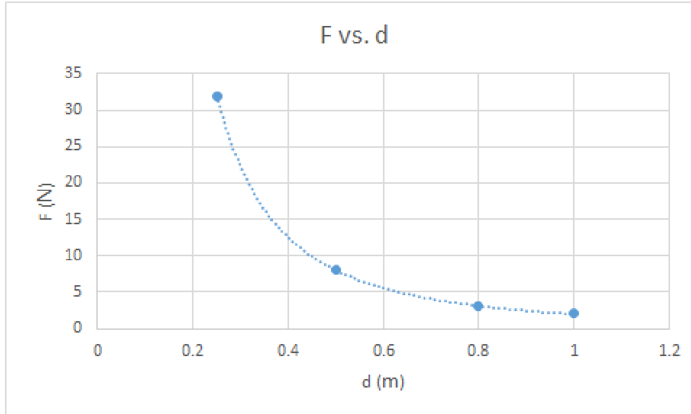
f) What is the **constant of proportionality (k)** for this data?

Check with your teacher.

EXAMPLE 2

Use the same method as above to determine the relationship between B and A:

d (m)	F (N)
0.25	32
0.50	8.0
0.80	3.1
1.0	2.0



a) Type of Relationship: ________________
b) Guess the proportionality: _________________
c) Graph your guess: (remember, you only change the x column)

d) Correct Proportion: ___________________________
e) Equation relating F and d: [answer on next page]

f) What is the constant of proportionality (k) for this data?

Answer: $F=2Nm^2/d^2$ $k = 2Nm^2$

COMPARING TO THEORY

This is often the hardest part. When you do an experiment and look at the data, first you get a **proportionality** – for example $E \alpha v^2$. Then you get an **equation** – for example $E=2v^2$.

a) What is the constant of proportionality (k) for this data? _________

But now you have to compare it to a theoretical equation.

b) write the equation for kinetic energy: __________

How do we compare these? You must figure out what was constant. When you do an experiment to examine the relationship between two variables, all the other variables must remain constant.

c) In the example above, the experiment was between energy and speed. What must have remained constant?

So we could write the equation as $E = (½m)v^2$

d) What parts of the equation above make up the constant of proportionality? ________
e) So say the mass was 4.2 kg. What is the theoretical constant of proportionality?

f) Calculate the % error (or % deviation) between the experimental constant and the theoretical constant.

EXAMPLES

Fill in the table. The first one is done for you. Constants include units

Experimental Equation	**Other Quantities**	**Theoretical Equation**	**Theoretical k (symbols)**	**Value of Theoretical k**	**% Deviation**
$E=2kgv^2$	m=4.2 kg	$E = ½mv^2$ or $E = (½m)v^2$	½m	4.2 kg/2 = 2.1 kg	4.8%
$F = \frac{10Nm}{r}$	v=2m/s m=2.6 kg	$F = \frac{mv^2}{r}$			
$T = 2.1(\frac{s}{\sqrt{m}})\sqrt{L}$	g=9.81 N/kg	$T = 2\pi\sqrt{\frac{L}{g}}$			
$a = \frac{20m}{T^2}$	r=0.5m	$a_c = \frac{4\pi^2 r}{T^2}$			

PROPORTIONAL REASONING

When you know the relationship between two variables, you can make predictions about the variables, even if you don't have an equation.

Ex: You know that for a given time interval, the distance is proportional to the speed: $d \propto v$.

a) If the speed doubles, the distance will ________________________________

b) If the speed increases by a factor of 3, what happens to the distance travelled?

But things are different for different relationships. Consider once again, the equation for kinetic energy. If the mass of the object is 4.0 kg, fill in the table below:

v (m/s)	E_k (J)
1	
2	
3	
4	

a) When the speed doubles, what happens to the kinetic energy?

b) When the speed increases by a factor of 3, what happens to Ek?

c) When the speed increases by a factor of 4, what happens to Ek?

When v changes by a factor, energy changes by that factor squared. This is because the E_k is proportional to v^2 (not just v).

PRACTICE

1. If an object is accelerating, the time it takes to travel a distance travelled is proportional to the square root of the distance ($t \propto \sqrt{d}$). What happens to the time if the
   a. distance doubles?
   b. distance increases by a factor of 9?
   c. distance is reduced by a factor of 4?

Increases by 1.4; increases by a factor of 3; decreases by a factor of 2

SUMMARY – CREATE YOUR OWN SUMMARY OF TODAY'S LESSON:

HOMEWORK

1. It can be shown that the kinetic energy, E_k, of a moving car is proportional to the square of its velocity; in other words, $E_k \alpha v^2$. At 90 km/h, a certain car's E_k is measured as 400 units.
   a. What is the car's kinetic energy at 270 km/h?
   b. What is the car's kinetic energy at 45 km/h?
   c. What is the car's velocity when its kinetic energy is 16 units?
2. The force between any two charged particles is inversely proportional to the square of the distance between them; in other words, $F \alpha 1/d^2$. At a certain distance, the force is 36 N.
   a. What is the force if the distance is doubled?
   b. What is the force if the distance is reduced by a factor of 2?
   c. If the force is 4 N, what has happened to the distance?
3. For the equation $\Delta V = \frac{hcq}{\lambda}$, write the proportionality between
   a) ΔV and q
   b) ΔV and λ
   c) λ and q
4. For the equation in Q2, assume you do an experiment where you vary the values of λ and measure the resulting values of ΔV. What graph would you plot with your data to get a straight line?
5. Use the straightening the graph technique learned in this lesson to determine the equation that fits each set of data below. You may use LoggerPro to make your raw data graphs and straight graphs.

t (s)	d (m)
0.0	0.0
0.8	12.8
1.0	20.0
1.2	28.8
1.4	39.2

G (m/s)	H (N)
2.0	100
1.6	64
1.4	49
1.2	36
1.0	25

r (m)	F (N)
1.0	10
1.2	6.9
1.8	3.1
2.4	1.7
3.0	1.1

Answers:

1. a. 3600 units b. 100 units c. 18 km/h
2. a. 9 N b. 144 N c. tripled
3. a) $\Delta V \alpha q$ b) $\Delta V \alpha 1/\lambda$ c) $\lambda \alpha q$
4. see graph at the side
5. $d = (20 m/s^2)t^2$ $H = 25N/(m^2/s^2)G^2$ $F = \frac{10Nm^2}{r^2}$