the strobe rotates 24 times in 10.0 s. If this is the highest "stopping" frequency, what is the rate of rotation of the drill, in revolutions per minute? (1.73×10^3 r/min)

5. A ten-slit stroboscope was rotated at a constant rate for 20.0 s. It was used to determine that the period of a spinning motor shaft was 0.025 s. How many rotations did the stroboscope make during the 20 s interval? (80)
6. A six-slit stroboscope is rotated at the highest "stopping" frequency of 2.5 Hz for the blade of a food processor.
   (a) What is the period of the processor blade?
   (b) If every second slit on the strobe is taped closed, how fast must the strobe be rotated in order to be at the highest "stopping" frequency for the same food processor blade?
   (0.067 s, 5.0 Hz)

1.7 Analysing Experimental Data — Proportioning Techniques

In physics, we seek to correlate things we observe, that is, to see how a change in one quantity affects the value of another quantity. For example, how does the elapsed time affect the distance a car travels? Or, what is the relationship between the speed of a falling ball and the air resistance opposing the ball's motion? Or, how does the distance separating two masses affect the force of gravity between them? The stating of such relationships in a concise form is central to the structure of physics. The statements themselves become the laws of physics.

The statement of how one quantity varies in relation to another is called a **proportionality** expression. Most of the relationships described in this text are relatively simple, being either direct or inverse variations (proportions). An example of a direct proportion is the relationship between the distance travelled from the starting position and the time, for an object travelling at a uniform speed. As seen in the chart, when the time doubles, the distance doubles; when the time triples, the distance triples; and so on.

	×2	×3				×7	
time (s)	1	2	3	4	5	6	7
distance (m)	28	56	84	112	140	168	196
	×2	×3				×7	

Mathematically, we say that distance is directly proportional to time, or

$$d \propto t$$

Above and below the chart are arrows indicating that d's multiplier and t's multiplier are equal for the same pairs of numbers. This means that, if $d \propto t$, then

$$d\text{'s multiplier} = t\text{'s multiplier}$$

In the case of an inverse proportion, for example, the relationship between the frequency (f) and period (T) of a vibrating object, the multipliers are reciprocals of each other.

×2 ×10

frequency (Hz)	5	10	20	50	75	100
period (s)	0.2	0.1	0.05	0.02	0.013	0.01

×1/2 ×1/10

In this case, the frequency's multiplier must be inverted to get the period's multiplier. If $f \propto \frac{1}{T}$, then

$$f\text{'s multiplier} = \text{the reciprocal of } T\text{'s multiplier}$$

When a scientist attempts to determine how one measurable physical quantity (y) varies with another (x), he usually performs an experiment in which all other variables that might affect y are kept constant, and then he measures values of y for various values of x. The experiment thus yields a table of values, or a set of ordered pairs, for y and x, such as those listed in the sample problems, below. The scientist uses such tables to determine the relationship between y and x, asking the question, "What must I do to x's multiplier to get y's multiplier?"

Sample problems

1.

y	x
250	3
750	9
2500	30
5000	60

×3 ×3

×20 ×20

y's multiplier = x's multiplier

$\therefore$ y $\propto$ x

2.

	A	B	
	20	14	
$\times 4$	80	28	$\times 2$
	180	42	
$\times 100$	2000	140	$\times 10$

A's multiplier = B's multiplier squared

$\therefore A \propto B^2$

3.

	F	r	
	900	1	
$\times 1/4$	225	2	$\times 2$
$\times 1/25$	36	5	$\times 5$
	14	18	
$\times 1/900$	1	30	$\times 30$

F's multiplier = the reciprocal of r's multiplier squared

$\therefore F \propto \frac{1}{r^2}$

Practice

Determine the proportion for each of the following tables of values.

A	B
2	100
8	200
50	500
200	1000

C	D
3	120
6	60
9	40
12	30

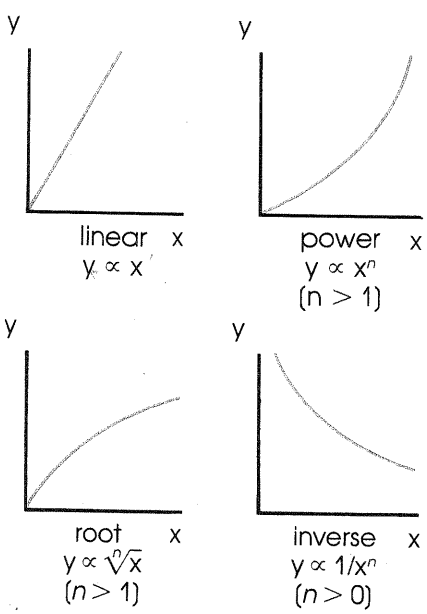
E	F
2	90
54	270
16	180
250	450

G	H
6	5
12	20
18	45
42	245

K	L
7	800
35	32
28	50
70	8

M	N
2	3
4	24
6	81
8	192

The examples used above involve easily calculated multipliers and ones that showed the relationship clearly. In practice, experimental results always involve some error, and the relationship may not be as obvious. In these cases, graphs can be used to determine the correct proportion.



First, plot y versus x. The variable on the left of the proportion is called the dependent variable, and it is usually plotted on the vertical axis. Four basic graph types are illustrated.

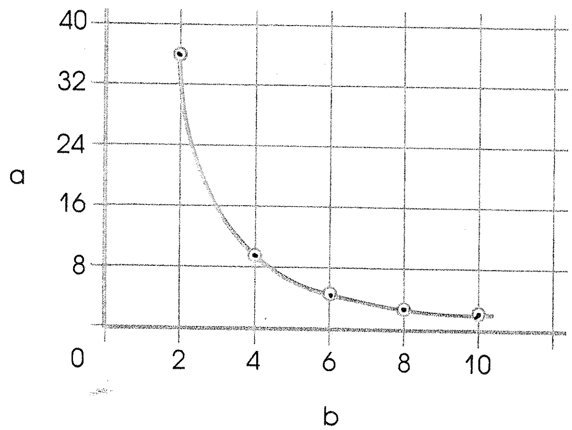
Note that the only graph that is a straight line and passes through zero is $y \propto x$. Thus, if when the experimental values are plotted they form a straight line through the origin, the proportion must be $y \propto x$. If the graph produced is any one of the other shapes, we can tell the general form of the proportion but we do not know the value of "n".

To determine the value of n, we can replot y versus some function of x that we choose. For example, if y versus x appears to be a power curve ($y \propto x^n$), we might try plotting y versus x^2 or y versus x^3. The correct choice will produce a linear graph for y versus the correct power of x. If, for example, a graph of y versus x^3 is a straight line through the origin, then $y \propto x^3$.

Unfortunately, this is a trial-and-error process, but we can often make an educated guess on the basis of the multipliers of the two variables or from our understanding of the physics underlying the investigation. In general, when y versus x^n makes a linear graph, $y \propto x^n$.

Sample problem

Given the table of experimental results below, find the relationship between a and b.

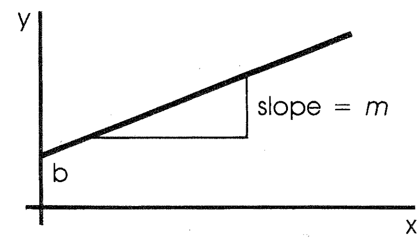


b	a
2.0	36
4.0	9.0
6.0	4.0
8.0	2.3
10	1.4

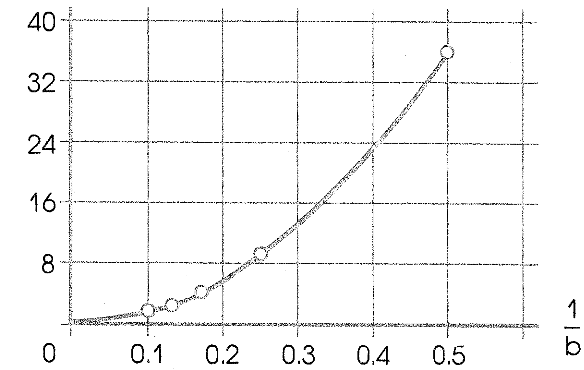
Plotting a graph of a versus b, we discover from its shape that a is inversely proportional to b^n ($a \propto \frac{1}{b^n}$).

Since $a \propto \frac{1}{b^n}$, we assume the simplest value of n, that is, $n = 1$, and set up a column of values of $\frac{1}{b}$. Then we plot a versus $\frac{1}{b}$.

We have limited our discussion here to relationships where the origin lies on the curve, since many of the relationships in physics, and most of those discussed in this text, have this condition. Relationships with a y-intercept are of the form $y = mx + b$, where m is the slope and b the intercept. Such a relationship will be encountered in Section 18.2.

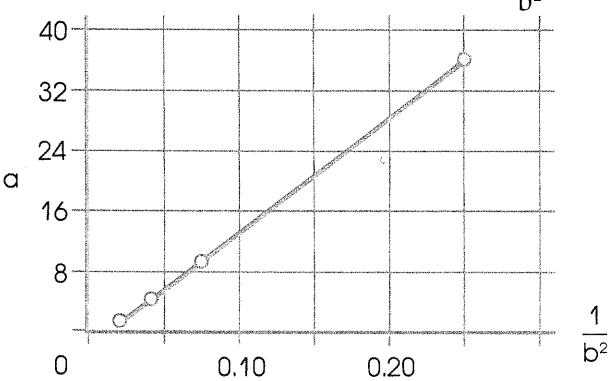


$\frac{1}{b}$	a
0.50	36
0.25	9.0
0.17	4.0
0.13	2.3
0.10	1.4



Since the plot of a versus $\frac{1}{b}$ does not produce a straight line, we set up a column of values for $\frac{1}{b^2}$ and plot a versus $\frac{1}{b^2}$. As illustrated below, this plot does produce a straight line, and thus $a \propto \frac{1}{b^2}$.

$\frac{1}{b^2}$	a
0.25	36
0.063	9.0
0.019	4.0
0.017	2.3
0.010	1.4



Practice

1. In the following table of results, determine the relationship between F and r, using graphical techniques.

r	1	1.2	1.8	2.4	3.0
F	10	6.9	3.1	1.7	1.1

2. A slider that starts from rest and slides down an inclined air track covers the distances d in the times t. Using graphical methods, determine the relationship between d and t.

t	0	0.8	1.0	1.2	1.4
d	0	12.8	20.0	28.8	39.2

3. An experiment is performed to find the relationship between two physical quantities, B and A. The following data is obtained.

A	100	64	49	36	25	16
B	1.99	1.59	1.39	1.19	1.00	0.80

Determine the relationship between B and A.

1.8 Using Proportioning Techniques in Physics

Forming Equations from a Proportion

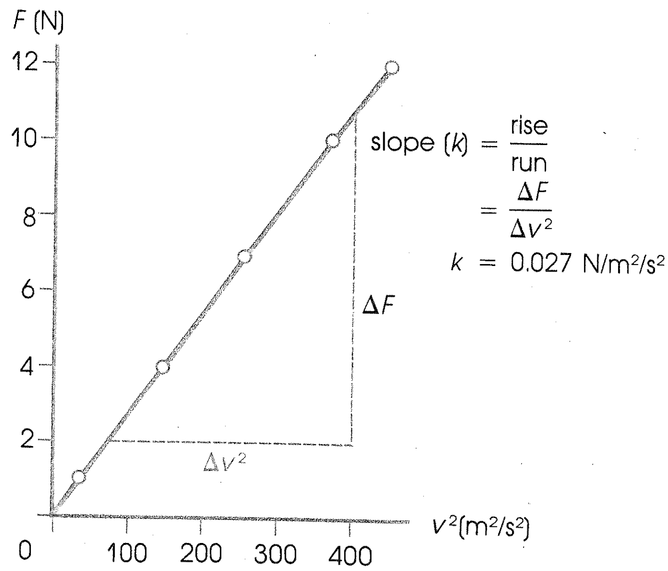
Once the proportionality has been determined, the next step is usually to change it into an equation. This makes it possible to link the two quantities numerically. To form an equation from a proportion, simply replace the proportionality sign ($\propto$) by an equals sign ($=$) and a proportionality constant (usually "k").

Thus, $y \propto x^n$ becomes $y = kx^n$

The preferred method for determining the value of the proportionality constant is to find the slope of the straight-line graph relating the two variables. Suppose, for example, that it is found that two variables, F and v, are related by the proportionality statement $F \propto v^2$. A graph of F versus v^2 produces a straight line, as illustrated. The slope of the graph provides the numerical value of k, in this case 0.027 $N/m^2/s^2$. When substituted back into the general equation, it becomes $F = 0.027\ v^2$. This equation will be valid only for F in newtons and v in metres per second.

The units for k, $N/m^2/s^2$, can also be expressed as $N \cdot m^{-2}s^{-2}$.

F(N)	v(m/s)	v^2(m²/s²)
1.0	6.0	36
4.0	12.0	144
7.0	16.0	256
10.0	19.2	369
12.0	21.1	445



In mathematics, the value of the constant "k" is usually determined by substituting in only one ordered pair. This is a dangerous procedure in physics. All measured data contain error. The ordered pair substituted in the equation could possibly be the least precise pair of values. The resulting equation will not accurately describe the relationship between the two variables.

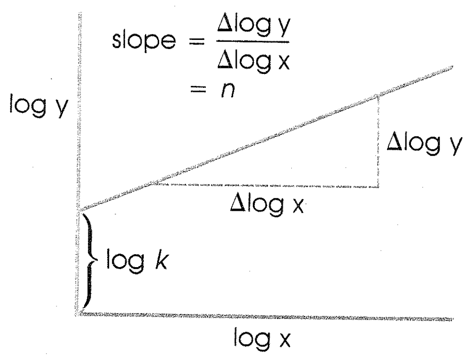
If it is not practical to draw a graph, an average, or "best", value for the proportionality constant may be calculated, using the pairs of values from the experimental data. For example, if $I \propto \frac{1}{d^2}$, then $I = \frac{k}{d^2}$, or $k = Id^2$. If two measured values are $d = 3.0$ cm and $I = 10$ lx, then

$$\begin{aligned} k &= Id^2 \\ &= (10\ \text{lx})(3.0\ \text{cm})^2 \\ &= 90\ \text{lx}\cdot\text{cm}^2 \end{aligned}$$

This procedure is repeated for the other pairs of values, and an average value of k, for example, 92, is calculated. The resulting equation, $I = \frac{92}{d^2}$, is valid only for I in lx and d in cm, since these are the units that were used to calculate k. Some electronic calculators are programmed to do the above calculation or some other line-fitting method, displaying the average value of k as the slope of the graph.

The lux (lx) is the SI unit of illuminance.

Using Logarithms to Determine an Equation



Note that the equation $\log y = \log k + n\log x$ may be rewritten as $\log y = n\log x + \log k$, which is the same form as $y = mx + b$.

The general proportion, $y \propto x^n$, expressed as an equation, is $y = kx^n$. Taking the logarithm of each side of the equation, it may be rewritten as

$$\log y = \log k + n\log x$$

When the logs of the values of x and y are plotted in a graph of log y versus log x, a straight-line graph results, as illustrated. The slope of this graph is n, and the log y intercept is log k. The values of n and k are then substituted back into $y = kx^n$ to determine the equation.

Forming a Proportion from an Equation

The proportional relationship may be formed from an equation by working backwards from the equation. All numerical constants, and any variable that is held constant for the purpose of the problem, are replaced by a single constant, k. Then the equals sign and the constant, k, are replaced by the proportionality sign.

For example, one equation for centripetal acceleration (Section 3.9) is $a_c = \frac{4\pi^2 R}{T^2}$ or $4\pi^2 R \times \frac{1}{T^2}$

but $4\pi^2 R$ = a constant, k, if R is kept constant.

Therefore, $a_c = k \times \frac{1}{T^2}$

and $a_c \propto \frac{1}{T^2}$ (if R is held constant).

Similarly, $a_c \propto R$ (if T is held constant).

Solving Problems Using Proportioning Techniques

If the proportion relating two variables is known, many seemingly difficult problems can be solved easily, using proportioning techniques. A series of sample problems will illustrate different applications of these techniques.

Sample problems

1. The force of air resistance (F) on a moving body is related to the speed of the body (v) by the proportion $F \propto v^2$. If the speed triples, how many times greater is the force?

Since $F \propto v^2$
F's multiplier = v's multiplier squared
$F' = F \times 3^2$
or $F' = 9\ F$ (F' means the new value of F)

Thus, if the speed triples, the force increases by a factor of nine.

2. A cylindrical water tank holds 1.0×10^5 L of water. How much would it hold if all of its dimensions were doubled? The volume of a cylinder is given by

$$V = \pi r^2 h$$

Since all of the linear dimensions (l) are doubled and π is a constant, the equation may be expressed as a proportionality, i.e., $V \propto l^3$.

$$V' = V \times 2^3$$
$$= (1.0 \times 10^5\ \text{L})\ (8)$$
$$V' = 8.0 \times 10^5\ \text{L}$$

The same would be true for *any* shape of tank, as long as the shape did not change when the dimensions were scaled up or down.

3. The equation describing the displacement of a uniformly accelerating object starting from rest is $d = 0.5at^2$, where d is the displacement, a is the acceleration, and t is the time interval. An object travels 100 m in 10 s when its acceleration is 2.0 m/s². What is its displacement if the acceleration is 0.50 m/s² and the time interval is 40 s?

$d_1 = 100$ m $\quad a_1 = 2.0$ m/s² $\quad t_1 = 10$ s
$d_2 = ?$ $\quad a_2 = 0.50$ m/s² $\quad t_2 = 40$ s

In this question, acceleration is multiplied by a factor of 1/4 and time is multiplied by a factor of 4.

Since $d \propto at^2$ (from the equation $d = 0.5at^2$)
a is multiplied by a factor of 1/4
t is multiplied by a factor of 4 and
t^2 is multiplied by a factor of 16
so d_1 is multiplied by a factor of $1/4 \times 16 = 4$
therefore $d_2 = d_1 \times 4$
$= 100\ \text{m} \times 4$
$= 400$ m

Alternate Solution

Since $d = 0.5at^2$, $d \propto at^2$

therefore $\dfrac{d_1}{d_2} = \dfrac{a_1 t_1^2}{a_2 t_2^2}$

$$\frac{100\ \text{m}}{d_2} = \frac{(2.0\ \text{m/s}^2)(10\ \text{s})^2}{(0.5\ \text{m/s}^2)(40\ \text{s})^2}$$

$$d_2 = 400\ \text{m}$$

Practice

1. Express each of the following equations as proportions, using only the variables indicated.
   (a) $V = 4/3\pi r^3$; V and r
   (b) $F_c = \frac{mv^2}{R}$; (i) F_c and v, (ii) F_c and R
   (c) $F_g = \frac{Gm_1m_2}{R^2}$; (i) F_g and m_1, (ii) F_g and R, where G is a constant
   (d) $K = \frac{R^3}{T^2}$; R and T, where K is a constant
2. Given the relationship $E \propto mv^2$: (a) If E is 98 units when m is 4.0 units and v is 7.0 units, express the proportion as an equation.
   (b) What is the value of E when m is 10 units and v is 42 units? (8.8×10^3)
3.

x	0.2	0.4	0.6	0.8	1.0
y	200	50	22.2	12.5	8.0

   (a) Determine the proportion relating y and x.
   (b) Write an equation relating y and x.
   (c) If x = 0.55, what is y, predicted from (b)? (26)
4. Two neighbours have swimming pools with identical shapes. One pool holds 2.0×10^4 L of water. How many litres will the second pool hold if all its dimensions are 1.6 times as large? (8.2×10^4 L)
5. Given $F_c = \frac{mv^2}{R}$
   What is the effect on F_c of each of the following?
   (a) increasing m by a factor of 3
   (b) decreasing v to 1/3 of its former value
   (c) decreasing R to 1/4 of its former value
   (d) all of the above ($\times$ 3, $\times$ 1/9, $\times$ 4, $\times$ 4/3)
6. If $a \propto b^3$ and a = 4.0 when b = 3.5, what is a when b = 7.0? (32)
7. Given that $d \propto at^2$. If a = 2.0 m/s² and t = 4.0 when d = 32 m, what will be the value of d when a = 4.0 m/s² and t = 12 s? (5.8×10^2 m)
8. Given that $p \propto q^3/r^2$, and that p = 400 when q = 5.0 and r = 3.0. Calculate the value of p when q = 15 and r = 5.0. (3.9×10^3)